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SOLOW NEOCLASSICAL ECONOMIC GROWTH MODEL: PROPOSAL OF A NEW APPROACH

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Abstract: The neoclassical Solow (1956) model is, non-arguably, a universal reference in term of exogenous economic growth model. Nonetheless, an important potential of this model could be unleashed if a different mathematical step from what is commonly known is adopted, in the case of technical progress, as well as in the case of presence of human capital. In both cases, it appears more clearly that Technical Progress and Human Capital, which are exogenous, in the sense that they are independant of the physical capital, are contributing in the effort of compensating, and even, overtake negative effects of both demographic expansion and depreciation rate, and their effect may eventually results in eliminating the steady state, therefore allowing, so to speak, the economy, to grow indefinitely.

This article reinforce the role of technology, specifically, and of knowledge in general, in triggering economic growth and changes in developping countries, which are generally struggling with high demographic growth rates and poorly maintained infrastructures.

Keywords: Solow model, technical progress, human capital.

Résumé : Le modèle néoclassique de Solow (1956) est, incontestablement, une référence universelle en terme de modèle de croissance économique exogène. Néanmoins, un potentiel important de ce modèle pourrait être libéré si une démarche mathématique différente de ce qui est communément connu est adoptée, dans le cas du progrès technique, ainsi que dans le cas de la présence de capital humain. Dans les deux cas, il apparaît plus clairement que le Progrès Technique et le Capital Humain, qui sont exogènes, au sens où ils sont indépendants du capital physique, contribuent à l'effort de compensation, et, même, de dépassement, des effets défavorables à la fois de l'expansion démographique et du taux de dépréciation, et leur effet est susceptible, *in fine*, d'entraîner l'élimination de l'état stationnaire, permettant, pour ainsi dire, à l'économie, de croître indéfiniment.

Le présent article confirme le rôle de la technologie en particulier et de la connaissance en général, comme étant des facteurs principaux d'accélération de la croissance écoçnoique et du changement pour les pays pauvres, en proie à un taux de croissance démographique important et à un problème de maintenance au niveau des infrastructures.

Mots-Clés : Modèle de Solow, progrès technique, capital humain.

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1. Introduction

All economists must have come accross this model which central idea says that diminishing returns to investments implies that the economic growth declines as it approaches the steady state of capital per unit of effective labor. It was critical in demonstrating and establishing importance of technological progress and human capital in generating long term economic growth. In the Solow model, technology is exogenous, in the sense that it is independant of capital. Additionally, it is commonly said as well that technology comes from nowhere.

In our work, we mathematically turned differently variables in the situation where the model integrates technical progress, and another one integrating both technology and human capital. This proposed approach resulted in radically different results when compared to Solow's results, but it is provided with the advantage of being more coherent with intuitive expectation.

I. The Well-Known Solow Elementary Model

The model is traditionnally presented as being composed by two equations : $Y=K^{\alpha}L^{1-\alpha}$ $(0<\alpha<1)$, which is the production function, and $\dot{K}=sY - \delta K$, which is the capital accumulation equation. s is the saving, and δ is the depreciation rate. If we allow $y=\frac{Y}{L}$, $k=\frac{K}{L}$, and $n=\frac{\dot{L}}{L}$ (demographic growth rate), then, through a logarithmic derivation of $k=\frac{K}{L}$ we have $\frac{\dot{k}}{k}=\frac{\dot{K}}{K}-n$. Hence, $\dot{k}=sy-(\delta+n)k$. This gives us the following Solow diagramme :

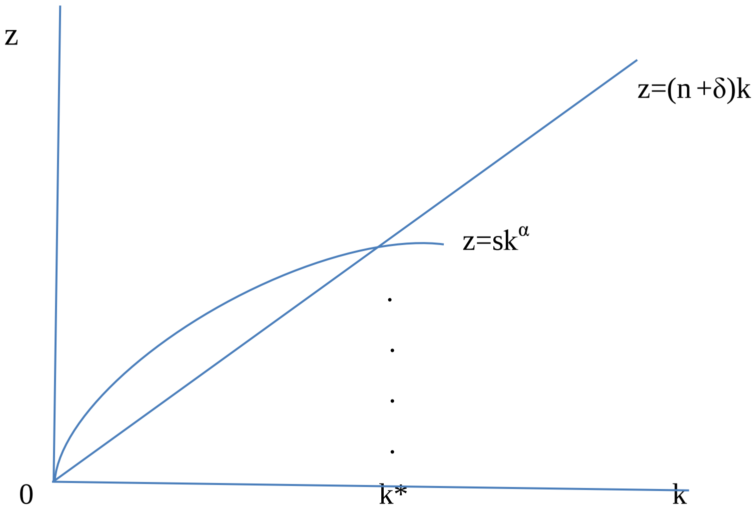

Figure 1 : Solow diagramme associated with the Solow basic model

Let's look more closely at this diagramme, and do what is called a « static comparative ». If we call k* the value of k corresponding to $\dot{k}=0$, we have to understand that at k*, the economy is steading. Growths of n and δ decrease the value of k*, meaning that the steady point is reached quicker. n and δ have, therefore, to be understood as being « hostile » to economic growth. Now, if we increase the value of the saving s, k* is reached later, meaning that s is, in contrary, favorable to growth. It is easy to establish $k^*=(\frac{s}{n+\delta})^{1/(1-\alpha)}$. The production per capita is, therefore, $y^*=(k^*)^{\alpha} = (\frac{s}{n+\delta})^{\alpha/(1-\alpha)}$. We notice that, once k* is reached, $\dot{k}$ starts to be negative, which means that the economy starts to decrease, whatever the value of k is. This has a critical consequence on poor countries development strategy. Indeed, first, the only one way to avoid reaching the steady-point, and, thus, maintain intense growth in technologically poor countries is by diminishing both depreciation and demographic growth rate[1], and by increasing saving. Results are, from now, going to radically differ from what is obtained in the Solow theory.

2. Our Proposed Different Approach for the Solow Exogenous Growth

2.1 The Model with Technical Progress

Let's, now, indeed, introduce technology in the Solow model. It is important to notice, at this stage, that technology or technical progress, here, has to be understood as being the factor which is enhancing the physical capital. For mathematical convenience, we'll call A^{α} the technology factor, which value is obviously greater than 1. If $Y=K^{\alpha}L^{1-\alpha}$ (III.1), we'll let $\tilde{Y} = A^{\alpha}K^{\alpha}L^{1-\alpha} = A^{\alpha}Y$ (III.1'), meaning, therefore, that $Y=A^{-\alpha}\tilde{Y}$ (III.1''). The different approach we propose is to set $\tilde{Y} = (AK)^{\alpha} L^{1-\alpha}$, instead of $K^{\alpha}(AL)^{1-\alpha}$, as used in the Solow theoretical approach. We, then, let $\tilde{K}=AK$, which implies $\tilde{Y} = \tilde{K}^{\alpha}L^{1-\alpha}$, and $K=A^{-1}\tilde{K}$ (III.2). If we set $\tilde{k} = \frac{\tilde{K}}{L}$, then a logarithmic derivation of $\tilde{k} = \frac{AK}{L}$ gives $\frac{\dot{\tilde{k}}}{\tilde{k}} = \frac{\dot{K}}{K} + \frac{\dot{A}}{A} - \frac{\dot{L}}{L}$. If we set $g = \frac{\dot{A}}{A}$, the technical progress growth, we now have $\frac{\dot{\tilde{k}}}{\tilde{k}} = \frac{\dot{K}}{K}+g-n$ (III.3). Let's, then, consider the capital accumulation equation, $\dot{K} = sY - \delta K$ (III.4). If we divide the two members by K, we have $\frac{\dot{K}}{K} = s\frac{Y}{K} - \delta$, which gives $\frac{\dot{K}}{K} = s\frac{Y/L}{K/L} - \delta$ (III.4'). Hence, if we combine (III.1''), (III.2), (III.3) and (III.4'), we obtain $\frac{\dot{\tilde{k}}}{\tilde{k}} - g + n = s\frac{A^{-\alpha}\tilde{y}}{A^{-1}\tilde{k}} - \delta$, which is equivalent to $\dot{\tilde{k}}=sA^{1-\alpha}\tilde{y}-(n+\delta-g)\tilde{k}$. After checking that $\tilde{y}=\tilde{k}^{\alpha}$, we, ultimately, get $\dot{\tilde{k}}=sA^{1-\alpha}\tilde{k}^{\alpha}-(n+\delta-g)\tilde{k}$ (TP). What we have here seems to be more coherent with the fact that, intuitively, exogenous technology has to compensate the

[1] It explains, for instance, why China adopted the « one-child policy », a rather controvesial birth rate reduction policy which was implemented in this country, from 1979 to 2015.

advserse effects of both demographic and depreciation rates on economic growth. If we, indeed, pay a close attention to this (TP) equation, we notice that the expression n+δ–g, because of the negative sign of the technology rate g, radically differs from the expression n+δ+g which has always been associated with the Solow model with technical progress. Diagrammes associated with this result can vary, according to the value of g, compared to n+δ.

<u>Case 1</u> : g<n+$\boldsymbol{\delta}$. In this situation, in which g doesn't totally compensate the value of n+δ, the economy, eventually, reach a steady state once $\tilde{k}$= $\tilde{k}$*, a point where $\dot{\tilde{k}}$=0.

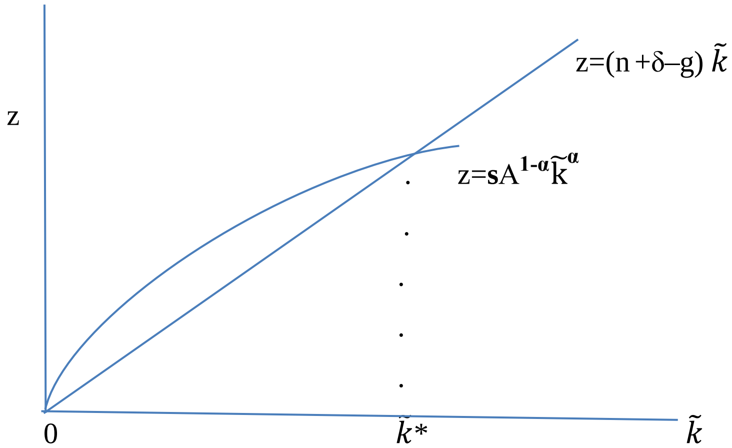

Figure 2 : Solow diagram associated with the case with technical progess, with g<n+$\boldsymbol{\delta}$.

It is easy to check that $\tilde{k}* = (\frac{sA^{1-\alpha}}{n+\delta-g})^{1/(1-\alpha)}$, which does exist as long as the term $n+\delta-g$ differs from 0. The corresponding production per capita is $\tilde{y}* = \tilde{k}^{*\,\alpha} = (\frac{sA^{1-\alpha}}{n+\delta-g})^{\alpha/(1-\alpha)}$. To finalize, we need to remember that y=$A^{-\alpha}\tilde{y}$, meaning that y*=$A^{-\alpha}(\frac{sA^{1-\alpha}}{n+\delta-g})^{\alpha/(1-\alpha)}$, or, finally, $y* = (\frac{s}{n+\delta-g})^{\alpha/(1-\alpha)}$. At this point, let's make a few comments :

1°) In the case A=1, which means that the economy is not provided with technology, we have g=0, and, therefore, we retrieve the elementary version of Solow model.

2°) The curve of the function z=sA$^{1-\alpha}\tilde{k}^{\alpha}$ is as it is, since A is an exogenous factor, and, therefore, independent of $\tilde{k}$.

3°) From this value $\tilde{k}$* of $\tilde{k}$, $\dot{\tilde{k}}$<0, *i.e.* the economy starts decreasing.

4°) In the current presence of technical progress, a steady point does exist only in this case 1.

Let's now run, again, a comparative static. An increase of n or δ increases the slope of the line z=(n +δ–g) $\tilde{k}$, and increase the speed towards the steady point. On the opposite, an increase of g is slowing down the process towards $\tilde{k}$*.

<u>Case 2 :</u> g =n+$\boldsymbol{\delta}$.

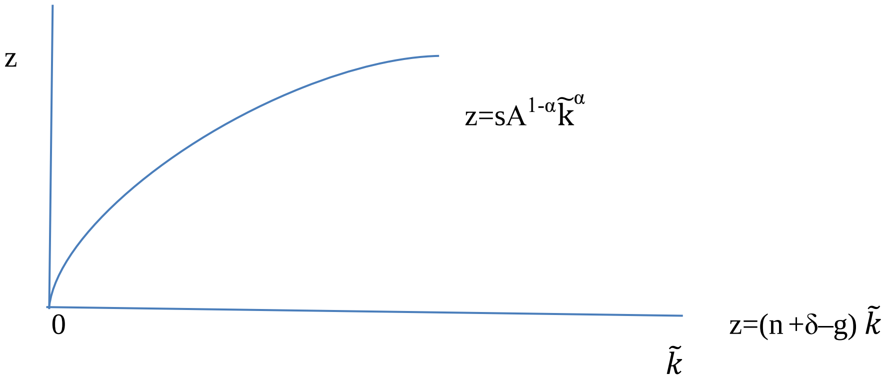

Figure 3 : Solow diagram associated with the case with technical progess, with g=n+$\boldsymbol{\delta}$.

In this case, the line z = (n + δ – g) $\tilde{k}$ is identified with $\tilde{k}$ axis. Economically, the value of g is, this time, able to compensate the negative effects of n and δ. In this situation, the steady point $\tilde{k}$* does not exist, and, as $\dot{\tilde{k}}$>0 permanently, the economy is indefinitely growing.

<u>Case 3</u> : g > n+$\boldsymbol{\delta}$. The line z=(n + δ – g) $\tilde{k}$ has a negative slope. Again, the steady point $\tilde{k}$* does not exist either, and the economy is, once again, permanently growing until n+δ overtakes g.

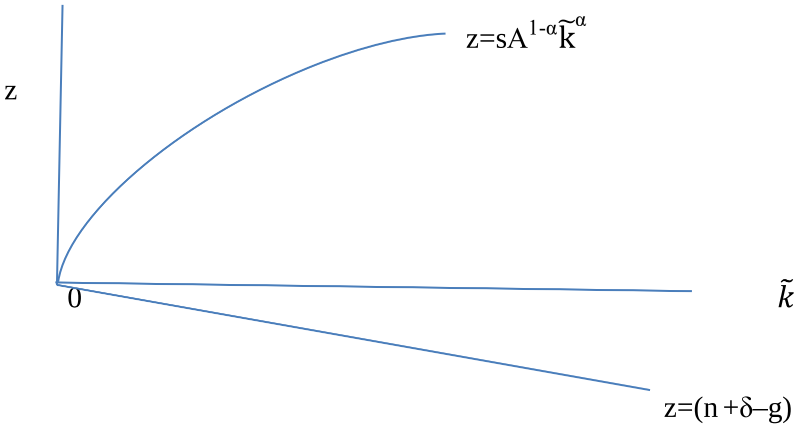

Figure 4 : Solow diagram associated with the case with technical progess, with g>n+$\boldsymbol{\delta}$.

2.2. Model with both Technical Progress and Human Capital

Now, we assume that the human capital, H = H(u) = L$e^{\psi u}$, where ψ is a constant linked to the economy, and u the average number of years spent at the university. We let h = H/L = $e^{\psi u}$, the human capital per capita. We notice that H(0) = L. As usually, let's start with the Solow elementary model composed by the two equations : $Y = K^{\alpha}L^{1-\alpha}$ (IV.1), and $\dot{K}$ = sY - δ K (IV.2). Since we are in a situation provided with both technical progress and human capital, we can assume that $\widehat{Y} = (AK)^{\alpha}H^{1-\alpha}$ (IV.1'). Economically, this means that A is a technological enhancing factor of the physical capital K, and H, i.*e.* the human capital, is an enhanced version of the labour L. (IV.1') is equivalent to $\widehat{Y} = (AKh^{(1-\alpha)/\alpha})^{\alpha}L^{1-\alpha} = (\widehat{A}K)^{\alpha}L^{1-\alpha}$, meaning that $\widehat{Y} = \widehat{A}^{\alpha}Y$, and $Y = \widehat{A}^{-\alpha}\widehat{Y}$, with $\widehat{A} = Ah^{(1-\alpha)/\alpha}$. After noticing the important fact that $\widehat{A}$ is playing, in (IV.1'), the same role as A in (III.1'), we can proceed to the logarithmic derivation of the equation $\widehat{A} = Ah^{(1-\alpha)/\alpha}$, and, thus, obtain $\widehat{g} = \frac{\dot{\widehat{A}}}{\widehat{A}} = \frac{\dot{A}}{A} + \frac{1-\alpha}{\alpha}\frac{h'}{h}$. If we set $a = \frac{\dot{h}}{h}$, then $\widehat{g} = g + \frac{1-\alpha}{\alpha}a$. Let's now, in the (TP) equation, replace A by $\widehat{A}$, g by $\widehat{g}$ and $\tilde{k}$ by $\widehat{k}$. We, consequently, have $\dot{\widehat{k}} = s\widehat{A}^{1-\alpha}\widehat{k}^{\alpha} - (n + \delta - \widehat{g})\widehat{k}$, which, ultimately, gives $\dot{\widehat{k}} = s\,A^{1-\alpha}\,h^{(1-\alpha)^2/\alpha}\widehat{k}^{\alpha} - (n + \delta - g - \frac{1-\alpha}{\alpha}a)\widehat{k}$ (HC & TP). Here again, we, have, for the same reasons as in III., three possibilities which can be summed up into two main scenarii :

- **<u>Case 1</u> : $g - \frac{1-\alpha}{\alpha}a < n+\delta$.**

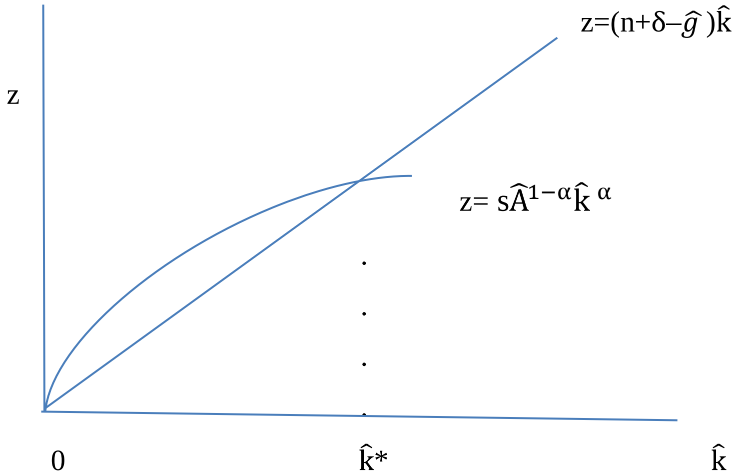

Figure 5 : Solow diagram associated with the case with technical progess and human capital, with $g - \frac{1-\alpha}{\alpha}a < n+\delta$.

The economy is going through a growing stage until it reaches it steady state at $\widehat{k}^*$. It is easy to check, from (HC&TP), that $\widehat{k}^* = \left(\frac{s\,A^{1-\alpha}\,h^{(1-\alpha)^2/\alpha}}{n+\delta-g-\frac{1-\alpha}{\alpha}a}\right)^{1/(1-\alpha)}$. Hence,

$\widehat{y}^* = \left(\frac{s\,A^{1-\alpha}\,h^{(1-\alpha)^2/\alpha}}{n+\delta-g-\frac{1-\alpha}{\alpha}a}\right)^{\alpha/(1-\alpha)} = \widehat{A}^{\alpha}y^*$. Consequently, $y^* = \left(\frac{s}{n+\delta-g-\frac{1-\alpha}{\alpha}a}\right)^{\alpha/(1-\alpha)}$.

- **<u>Case 2</u> : g - $\frac{1-\alpha}{\alpha}$ a ≥ n+δ**.

In this case, the value of g - $\frac{1-\alpha}{\alpha}$ a compensates the $(n + \delta)$'s « growth decreasing effect », and, even, offset $n+\delta$. As long as this situation prevails, we have $\dot{\hat{k}}>0$: the economy, at least theoretically, keeps growing and never gets into a steady state situation.

2.3. Theoretical Discussion

The well-known original version of the Solow model with technical progress uses an expression containing $n + \delta + g$. It always made me feel bad to think about it for at least two reasons. The first reason is that in this original version, it was always told that technical progress was contributing in making the economy to reach the steady state as well. This is, clearly, in contradiction with the very usefullness of technology, which is aimed at solving the puzzle of creating a long-term economic growth.

The second reason is that this original expression never gives a chance an economy to escape steady state thanks to neither technology or human capital.

The results we came with, through this different aproach have two consequences. The first one is that, for developing countries, who are not provided with a significant research and innovation capacity, long term economic growth may arise from technological transfer, especially if it is offered free of charge, or, at least, affordable. In the same line of thoughts, in order to grow, these countries need to establish a human-capital-development oriented partnership with more advanced economies. Significant increase of these two variables is the most important and efficient way to setting an economic configuration leading to recovering growth. I, consequently, call for a « knowledge diplomacy », which can be established in a win-win framework. In other words, international development agencies should focus more on organizing a massive technology and knowledge transfer from rich countries to poorest ones. That is, before they strengthen their inner innovation capacities, developing countries, if they are, indeed, meant to highly speed up their development process, need, first, to go through technological supports from international community, and this has to be accompanied by an internal natality reduction added to an infrastructure developement and depreciation avoidance effort. This will, then, incrementally, shift towards more endogenously generated technologies and growth.

Second, our theoretical results uncover the fact that use of highly evolved equipements has more impact on productivity, and, thus, on growth, than use of highly trained human capital. This induces the fact that technological development must be prioritized over everything, even over human capital forming, in order to generate growth in an economy. Another practial technical translation of this is that, along with creating an optmized business environment, developing countries have to create 4th generation special economic zones, which are including research and innovation centers. Last, this results may be understood as a profound necessity for research and development to focus more on creating technogically advanced production tools.

3. Conclusion

This approach shows a hidden potential of Solow's growth model and pays a special and deserved tribute to a particularly prolific and brightful work. My modest contribution was just to notice and bring this unexplored potential under the light, in order to be discussed and commented. One of the most important findings that stemed out of this paper is the fact that developing countries have to identify which technologies are the most fit to their development strategy, and they have then to train their human assets accordingly, provided that those are having the required technological absorption capacity.

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